Detection Performance of Radar Compressive Sensing in Noisy Environments

Asmita Korde[a],Damon Bradley[b] and Tinoosh Mohsenin[a]

[a]Department of Computer Science and Electrical Engineering, University of Maryland, Baltimore County

[b]NASA Goddard Space Flight Center

ABSTRACT

In this paper, radar detection via compressive sensing is explored. Compressive sensing is a new theory of sampling which allows the reconstruction of a sparse signal by sampling at a much lower rate than the Nyquist rate. By using this technique in radar, the use of matched filter can be eliminated and high rate sampling can be replaced with low rate sampling. In this paper, compressive sensing is analyzed by applying varying factors such as noise and different measurement matrices. Different reconstruction algorithms are compared by generating ROC curves to determine their detection performance. We conduct simulations for a 64-length signal with 3 targets to determine the effectiveness of each algorithm in varying SNR. We also propose a simplified version of Orthogonal Matching Pursuit (OMP). Through numerous simulations, we find that a simplified version of Orthogonal Matching Pursuit (OMP), can give better results than the original OMP in noisy environments when sparsity is highly over estimated, but does not work as well for low noise environments.

Keywords: Radar Compressive Sensing, ROC curves, OMP, Simplified OMP, CoSaMP, IHT, L1 regularized least squares

1. INTRODUCTION

In the recent years, Compressive sensing (CS) has emerged as a new technology to reconstruct sparse signals. In traditional methods, a signal has to be sampled at the Nyquist rate in order to reconstruct the signal without aliasing. However, if a signal is sparse, then obtaining Nyquist rate samples are wasteful. An alternative to Nyquist rate sampling would be to under-sample. Down-sampling or under-sampling a signal can lead to a loss of information especially if the signal is sparse. A new approach to this type of problem is compressive sensing. Instead of obtaining a few samples of the signal, a few linear combinations are sampled. Hence, one measurement consists of a collective sum.[1−2] Using fewer measurements, a signal can be reconstructed almost perfectly under certain conditions. In radar, CS can be used to determine probability of detection and false alarm. This helps in understanding the effectiveness of a CS radar detector. The goal of this paper is to compare the detection performance of different CS based reconstruction algorithms. To begin with, we first provide background on some CS technqiues as well as its application to radar, followed by a brief explanation of the reconstruction algorithms we are going to use. These algorithms include L1 regularized least squares, Iterative Hard Thresholding (IHT), Orthogonal Matching Pursuit (OMP), Compressive Sampling Matching Pursuit (CoSaMP), and Simplified Orthogonal Matching Pursuit. Then, we compare and analyze the detection performance of the reconstruction algorithms in very noisy as well as moderately low noise environments.

2. BACKGROUND

2.1 Compressive Sensing Theory

The theory behind compressive sensing lies in solving equation 1.

$$y = \phi x \tag{1}$$

Let us assume x to be a k-sparse signal of length N. Let ϕ be the measurement matrix projected onto the original signal, x. ϕ must be incoherent with the basis of the sparse signal, x. If x is not sparse in its original bases, it can be transformed to another domain in which the signal is sparse. Then the measurement matrix has to be

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uncorrelated with the signal in the transformed domain.[1−3] The size of ϕ is MxN, where $M<<N$ and represents the number of measurements. y is a M length vector containing the measurements obtained by the projection of ϕ onto x.

2.2 Radar CS

In Radar Compressive sensing, the transmitted signal must be incoherent with some basis. We can create projections of this incoherent signal on to a sparse target reflectivity profile. By generating only a few (less than the Nyquist rate) projections, we can reconstruct the target reflectivity profile.By using CS technique in radar, the use of matched filter can be eliminated and sampling can be done at a low rate.[4−6] Although data acquisition is a much efficient process in CS based radar, the reconstruction of the signal can be computationally complex. Hence, the key is to use an algorithm which balances between good detection performance and low computational cost. Radar based CS can be very beneficial in satellite or space communication, where data transmission is limited due to bandwidth. By using radar CS, we can reduce the amount of data transmitted and get a good reconstruction of the signal back. Since reconstruction can be done on the ground, power can be saved for on board communication.

2.3 Receiver Operating Curves (ROC)

ROCs are used to determine the effectiveness of a radar detector. They are generated by plotting probability of detection versus probability of false alarm. Once CS techniques are performed on the original signal, we can derive the estimate by a reconstruction algorithm. For each algorithm a different parameter is changed in order to change the sparsity of the signal. The estimates are compared to the original target profile and probability of detection and false are obtained.[7−8]

Detection:

$$\widehat{x(i)} \neq 0 \; and \; x(i) \; \neq 0$$

$$where \; \widehat{x} \; is \; the \; estimated \; signal$$

$$and \; i \; is \; the \; i^{th} \; element \; of \; the \; signal$$

Probability of detection:

$$pD = \frac{number \; of \; Detected}{number \; of \; x(i) \; \neq 0}$$

False Alarm:

$$\widehat{x(i)} \neq 0 \; and \; x(i) = 0$$

Probability of False Alarm:

$$pFA = \frac{number \; of \; false \; alarm}{number \; of \; N - x(i) \; \neq 0}$$

$$x(i) \; \neq 0 \; can \; also \; be \; referred \; to \; as \; targets$$

Monte Carlo simulations are performed, and the average values of probability of false alarm and probability of detection are plotted in the ROC. In order to determine the ROC, we took 1000 Monte Carlo simulations for all our simulations.

2.4 Coherence between measurement matrix and original signal

The matrix projected onto the sparse target profile must be incoherent with the basis of the sparse signal. Coherence is the largest correlation between any two elements of the two signals. Hence, we want measurement matrices which are the most uncorrelated with the sparse signal. Random matrices are universally incoherent, and hence, make a good basis for the transmitted signal, or the measurement matrix.[2] In addition, if the signal is sparse in time domain, Fourier basis can also be used for the measurement matrix. In this section,we compare normal random (Gaussian) matrix, Bernoulli random matrix and Fourier matrix. Gaussian matrix give normal random variables, Bernoulli gives ± 1, and Fourier matrix gives a Fourier transform of the indices of the size of the matrix. 1000 Monte Carlo simulations are performed to find the maximum coherence between the measurement matrix and a sparse signal of length 64 with 3 targets of the same amplitude. The average coherence of all three measurement matrices are listed in Table 1.

Table 1. Coherence of Measurement Matrix

Measurement Matrix	Average Coherence
Gaussian	0.6436
Bernoulli	0.6172
Fourier	0.6212

In order for CS to give good results the measurement matrix and the sparse signal must be as incoherent as possible. From table 1 we can see that all three of them are highly uncorrelated since their correlation values are close to 0.

In Fig. 1, we compare the performance of the three measurement matrices listed in Table 1. The 64 length signal with AWGN of SNR 15 is reconstructed using OMP algorithm discussed in section 3.1.3. From Table 1, it can be noted that Bernoulli measurement matrix is the most uncorrelated with the original target profile, followed by Fourier and lastly by Gaussian. This is evident in Fig. 1. The best ROC is given by Bernoulli measurement matrix, followed by Fourier. Since Gaussian measurement matrix was the most correlated out of the three measurement matrices, the ROC performed slightly worse than the other two. Hence, we can see a direct relation between correlation and detection performance. In conjunction with Table 1, we can show that lower correlation of the measurement matrix basis and the sparse signal basis leads to a better reconstruction of the signal, and in turn, better detection in terms of radar.

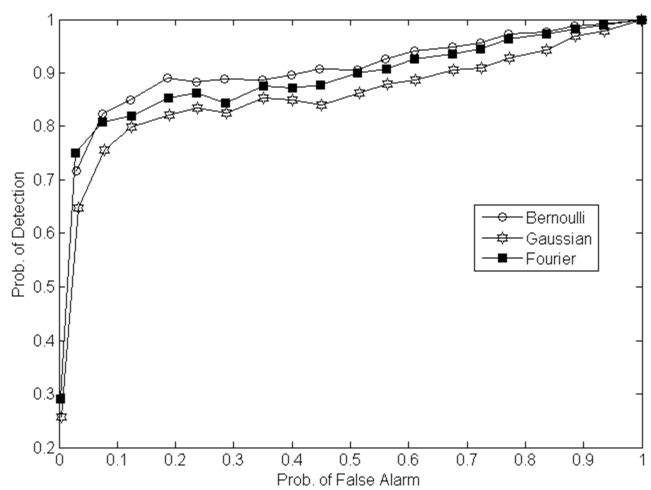

Figure 1. Comparison of measurement matrices for a 64 length signal with 3 targets and AWGN of SNR 15.

3. RECONSTRUCTION ALGORITHMS

Once we obtain low-rate samples for the receiver, we need to use a reconstruction algorithm to inversely solve for the original signal. We discuss below the reconstruction algorithms used in this paper.

3.0.1 L1 Regularized Least Squares

L1 regularized Least Squares solves for

$$minimize \parallel Ax - y \parallel_2^2 + \lambda \parallel x \parallel_1 \tag{2}$$

where λ is a regularization parameter. By changing the value of λ we can change the desired sparsity of the reconstructed signal. In order to solve for (2), truncated newton interior point method is used. The procedure shown below can be used to solve the newton's method.

We transform the problem into a convex quadratic problem. In our case, (2) would be transformed into

$$minimize \parallel Ax - y \parallel_2^2 + \lambda \sum_{i=1}^{n} u_i \tag{3}$$

subject to

$$-u_i \leq x \leq u_i \; i = 1, 2, , n \tag{4}$$

where

$$x \in R^n and \; u \in R^n \tag{5}$$

We solve the above convex quadratic problem using the truncated newton interior point method described in[9]. We will use the accompanied software package provided in[12] to perform the simulations in Section 4.

3.0.2 Normalized Iterative Hard Thresholding (IHT)

Normalized IHT is an iterative algorithm that uses a nonlinear operator to reduce the L0 norm at each iteration. The algorithm is described in detail in[11], which is based on the IHT algorithm described in[12]. This algorithm solves for:

$$x_t = argmin_x \; \|y - \phi_t x\|_2^2 + \lambda |x|_0 \tag{6}$$

$$x_0 = 0$$

$$g = \phi^T (y - \phi x^t) \tag{7}$$

$$\Gamma^n : \; Support \; set \; of \; x^t$$

$$Step \; size : \mu = \frac{g_{\Gamma^n}^T g_{\Gamma^n}}{g_{\Gamma^n}^T \phi_{\Gamma^n}^T \phi_{\Gamma^n} g_{\Gamma^n}} \tag{8}$$

$$x^{t+1} = H_k(x^t + \mu \phi^T (y - \phi x^t)) \tag{9}$$

Where H_k is the non linear operator, which sets all but the k max elements of $(x^t + \mu\phi^T(y - \phi x^T)) \; to \; 0$. From here on in the paper, we will refer to Normalized IHT as simply IHT. We will use the package provided in[13] for the simulations in Section 4.

3.0.3 Orthogonal Matching Pursuit (OMP)

OMP is a greedy algorithm- at each iteration, a column of ϕ is chosen which most strongly correlates with y. Then, the amount of contribution that the column provides to y is subtracted off. Further iterations are performed on this residual. After L iterations, the correct set of columns would be determined.[14] In this algorithm, we assume the desired sparsity to be k.
The algorithm is shown below:

$$1.Initialize\ R_0 = y,\ \phi_0 = \oslash, \Lambda_0 = \varnothing,\ \Theta_0 = \varnothing and\ t = 1$$
$$2.Find\ index\ \lambda_t = max_{j=1...n}\ subject\ to\ | < \phi_j R_{t-1} > | \tag{10}$$
$$3.Update\ \Lambda_t = \Lambda_{t-1} \cup \lambda_t \tag{11}$$
$$4.Update\ \Theta_t = [\Theta_{t-1}\ \phi_{\Lambda_t}] \tag{12}$$
$$5.Solve\ the\ Least\ Squares\ Problem:\ x_t = argmin_x\ \|y - \Theta_t x\|_2^2 \tag{13}$$
$$6.Calculate\ new\ approximation:\ \alpha_t = \Theta_t\ x_t \tag{14}$$
$$7.Calculate\ new\ residual:\ R_t = y - \alpha_t \tag{15}$$
$$8.Increment\ t,\ and\ repeat\ from\ step\ 2.\ if\ t < k$$

Once all the iterations are completed we hope to find the correct sparse signal x.

3.0.4 Compressive Sampling Matching Pursuit (CoSaMP)

CoSaMP is also based on OMP. This algorithm is described in more detail in[15]. It uses the columns of ϕ which produces the maximum dot product with y. The algorithm is shown below:

$$1.Initialize\ R_0 = y,\ x_0 = 0, \Theta_0 = \varnothing, and\ t = 0$$
$$2.Indentify\ 2k\ largest\ components:\ \Omega =\ supp_{2k\ j=1...n}\ subject\ to\ | < \phi_j R > | \tag{16}$$
$$3.Merge\ the\ supports: \Theta_t = [\Theta_{t-1}\ \Omega] \tag{17}$$
$$4.Solve\ the\ Least\ Squares\ Problem:\ a_t = argmin_a\ \|y - \Theta_t a\|_2^2 \tag{18}$$
$$5.Prune\ to\ retain\ k\ largest\ coefficients:\ x_t = a_{ts} \tag{19}$$
$$6.Calculate\ new\ residual:\ R_t = y - \alpha_t \tag{20}$$
$$7.Increment\ t,\ and\ repeat\ from\ step\ 2.\ if\ stopping\ criteria\ is\ not\ met$$
$$8.x = x_t \tag{21}$$

3.0.5 Simplified Orthogonal Matching Pursuit

We propose a simplified version of the OMP algorithm. Instead of choosing the columns of the new Θ by iterating through k times (where k corresponds to the sparsity), all the columns are chosen in one iteration itself.
The algorithm is described below:

$$1.Initialize\ R_0 = y,\ \phi_0 = \varnothing, \Lambda_0 = \varnothing$$
$$2.Find\ index\ set\ \lambda = supp_{k\ j=1...n}\ subject\ to\ | < \phi_j y > | \tag{22}$$
$$4.Let\ \Theta = [\phi_{\lambda_{1,..,k}}] \tag{23}$$
$$5.Solve\ the\ Least\ Squares\ Problem:\ x = argmin_x\ \|y - \Theta x\|_2^2 \tag{24}$$
$$6.Calculate\ new\ approximation:\ \alpha = \Theta\ x \tag{25}$$
$$7.Calculate\ new\ residual:\ R = y - \alpha \tag{26}$$
$$\tag{27}$$

The main difference between OMP and Simplified OMP is that Θ is not updated each time, but is formed all at once.

4. DETECTION PERFORMANCE

In this section we compare reconstruction algorithms for a 64-length signal with 3 targets of equal amplitude of value 1. We use 50 % of the measurements, i.e. 32 measurements to reconstruct the signal. 1000 monte carlo simulations are performed to attain the results shown later in this section. Since the same signal was used in Section 2.4, we found that Bernoulli measurement matrix works the best for this signal. Hence, bernoulli measurement matrix was used for all algorithms for all our simulations.

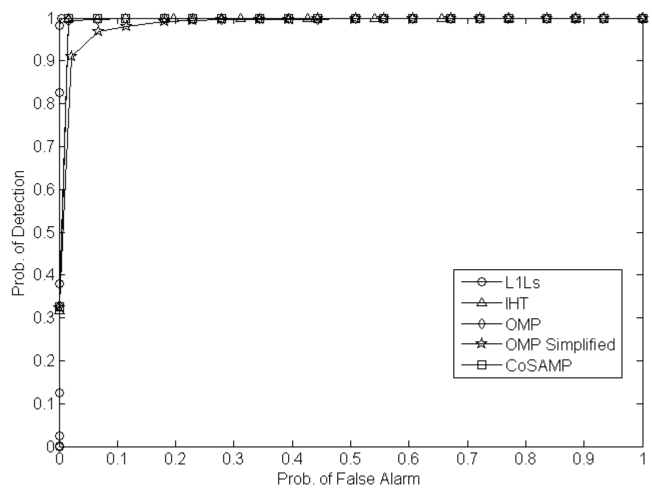

Figure 2. Comparison of different reconstruction algorithms for signal with AWGN of SNR 25

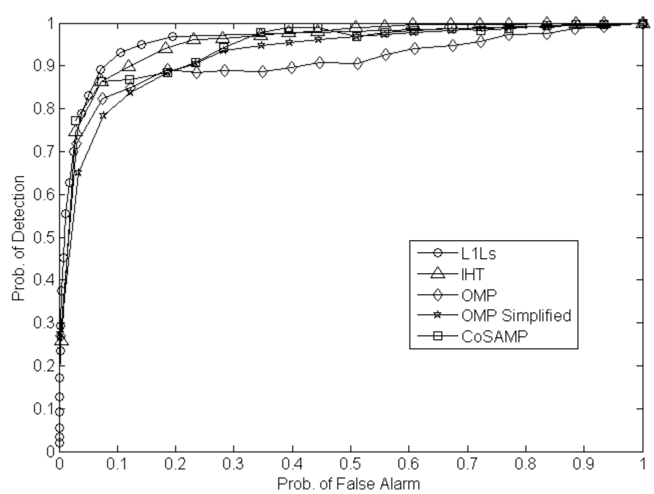

Figure 3. Comparison of different reconstruction algorithms for signal with AWGN of SNR 15.

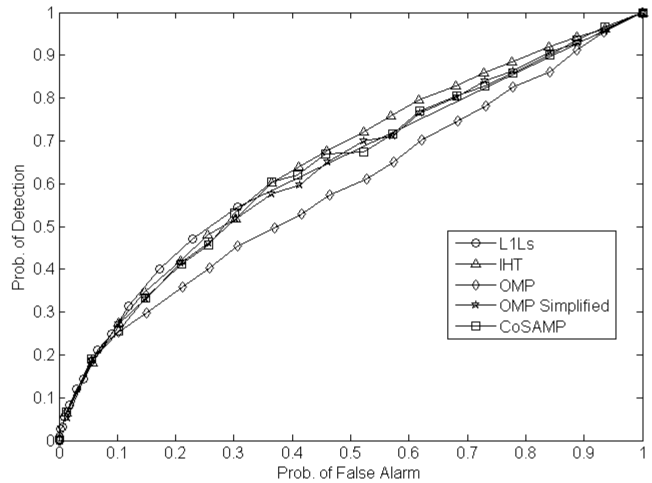

Figure 4. Comparison of different reconstruction algorithms for signal with AWGN of SNR 5

Furthermore, probability of detection was determined for varying SNR given a constant probability of false alarm of 0.2 for a 64- length target profile with 3 targets (Fig. 5). A very interesting find from the Fig. 2,3,4 and 5 is that OMP Simplified works better than OMP in noisy environments when sparsity is over estimated. As expected, we can see that L1 Regularized least squares and IHT worked better as compared to the variants of OMP. OMP Simplified worked better in low SNR as compared to OMP and vice versa.

L1 Regularized Least Squares and IHT work almost equally well. Normalized IHT is a relatively simple algorithm in comparison to L1 Regularized Least Squares. However, we should note that IHT can recover only very sparse signals, while L1 Regularized Least Squares work well for even moderately sparse signals.[16] L1 Regularized Least Squares requires to solve the newton method, which can be computationally very expensive. On the other hand, the computational complexity of OMP is low.[17−18] The most computationally expensive step in OMP is step 2 from the OMP algorithm mentioned above. The computation of the dot product can cost O(mN).[19] CoSaMP is a much faster variant of OMP, and can provide fairly well detection results compared to OMP as seen in figure 2,3 and 4. Similar to CoSaMP, simplified OMP is also a faster version of OMP since it does not have to go through k iterations and perform the dot product each time. By doing so, we can risk the accuracy of detection in moderately low noise signals (Fig. 2, 3 and 4). When the SNR is low, i.e, in a noisy environment, OMP simplified works as well as or even better than CoSAMP (Fig. 3, 4 and 5).

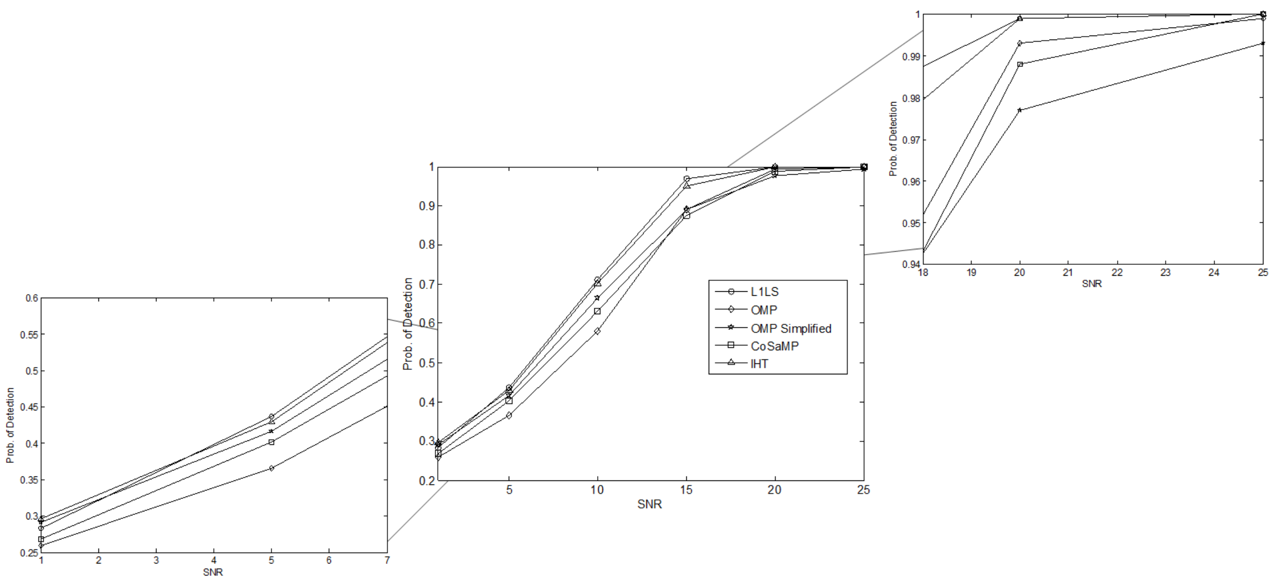

Figure 5. Comparison of different reconstruction algorithms for CFAR of 0.2.

In Table 2 we show the 95% confidence interval for SNR of 5 for all the reconstruction algorithms used in the figure above. Only the confidence interval for OMP does not overlap with the others and is distinctively low. The intervals for L_1 minimization and IHT are very close to one another and also lie in the higher range.

Table 2. 95 % Confidence Interval for Probability of detection for SNR 5

Reconstruction Algorithm	Upper Bound	Mean Probability Detection	Lower Bound
L1LS	0.4782	0.448	0.4178
IHT	0.4623	0.4297	0.3970
OMP	0.3823	0.3653	0.3483
CoSAMP	0.4241	0.4070	0.3899
OMP Simplified	0.4335	0.4163	0.3992

From Fig. 3, we can note that OMP and OMP Simplified cross over at a point. This shows that the estimated sparsity (under or over estimating sparsity) can make a difference in deciding if OMP outperforms OMP Simplified or vice versa. In Fig. 6, we show the effect of estimated sparsity and detection between OMP and OMP Simplified. For high SNR, such as SNR 25 shown in Fig. 6, OMP always has a higher detection performance until they even out at about a sparsity level of 30, where the detection is almost perfect. However, for lower SNR signals, OMP Simplified outperforms OMP if the sparsity is over estimated. For the plot in Fig. 6, OMP Simplified works better than OMP after the estimated sparsity level of about 13 and for SNR 5, after a sparsity level of 9. Hence, over estimating the sparsity of signal in noisy environments can lead to better detection performance by OMP Simplified as compared to OMP. However, overestimating sparsity can also lead to more false alarms.

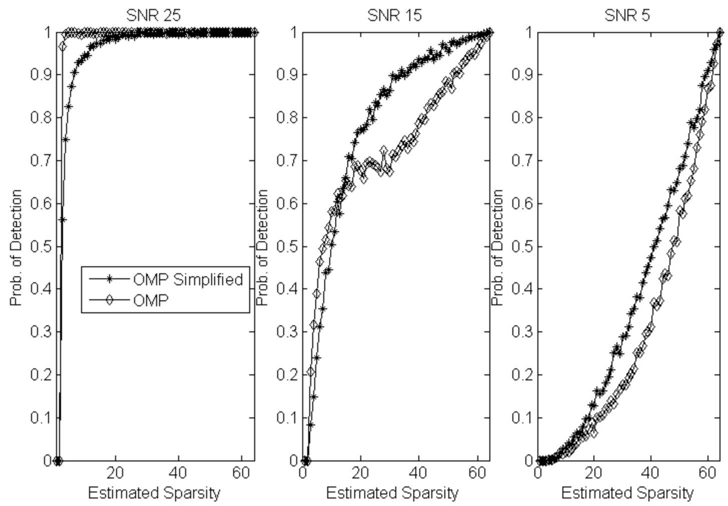

Figure 6. Comparison of detection perforamnce OMP and OMP Simplified for estimated sparsity. Left to right: SNR 25, SNR 15, SNR 5

5. CONCLUSION

As expected, L_1 minimization, IHT and CoSaMP are robust to noise. One significant finding of this work is that the OMP Simplified algorithm has a better detection performance as seen from the ROCs in noisy environments when sparsity is over-estimated, as compared to the performance of the regular OMP algorithm. Since OMP is a greedy algorithm, a mistake in the first iteration can cause significant error in finding the remaining columns. Moreover, OMP Simplified can work as well or even better than CoSaMP with signals with low SNR. Through our experiments, we are also able to show that L_1 minimization and IHT have a better detection performance than the variants of the greedy algorithm, OMP. Out of all the variants of OMP, CoSAMP has an overall better detection performance.

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